TECHNICAL NOTES

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No. 375

MOMENTS OF INERTIA OF SEVERAL AIRPLANES

By Marvel P. Miller and Hartley A. Soulé
Langley Memorial Aeronautical Laboratory

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Summary

The National Advisory Committee for Aeronautics has adopted the practice of measuring the moments of inertia of all airplanes that become available through their use in flight research work. This paper, which is the first of a series presenting the results of such measurements, gives the momental ellipsoids of ten army and naval biplanes and one commercial monoplane. The data were obtained by the use of a pendulum method, previously described. The moments of inertia are expressed in coefficient as well as in dimensional form, so that those for airplanes of widely different weights and dimensions can be compared. The coefficients are also useful for estimating the moments of inertia of airplanes for which no measurements of inertia have been made. To determine the accuracy with which the moments of inertia can be computed from design data, calculations were made of the moments of inertia for one of the above airplanes by summing up the moments of inertia of its constituent parts. It was found that the computed values were in error 20, 10, and 5 per cent for the X, Y, and Z axes, respectively.

Introduction

A careful quantitative study of the forces and the couples involved in the rotational motion of an airplane requires accurate data on its momental ellipsoid. An urgent need for such data appeared in connection with the study of spins being conducted by the National Advisory Committee for Aeronautics. The need was met by the development of an accurate experimental method for determining moments of inertia (Reference 1).

Because of the scarcity of reliable data on the subject, the Committee makes a practice of determining the moments of inertia of all airplanes that are used in its various researches. This paper presents the results of these measurements for eleven airplanes of various types and sizes. With the exception of one small commercial monoplane, the airplanes are representative army and naval biplanes.

The method used is the same as that described in Reference 1, with the difference that the virtual mass of the airplane, i.e., the actual mass plus an additional mass to account for the effect of the air disturbed by the motion of the airplane, is used in the calculations instead of the mass obtained by dividing the weight in air by the acceleration of gravity. Tests on a body of known moment of inertia have shown this method to give results with an error of less than 1 per cent.

As additional information of this kind is obtained, it will be made available through publication.

Results

The data for the airplanes investigated (Table I) include the moments of inertia and radii of gyration about the reference axes of the airplane and the angle of the principal axes with the reference axes. The sign of the angle of the principal axes indicates in which quadrants formed by the X and Z reference axes of the airplane the principal axes lie. Positive angles are measured counterclockwise in the plane of symmetry when the airplane is viewed from the left side. Since the X and Z reference axes are, by definition, assumed to lie in the plane of symmetry, the transverse axis is a principal axis. The X axis is taken parallel to the thrust axis.

A cross section of the momental ellipsoid for one of the airplanes is shown in Figure 1. This is an ellipse which has its major and minor axes inversely proportional to the radii of gyration K_X, K_Z about the principal axes of the airplane.

Nondimensional coefficients have been calculated for the purpose of comparing the moments of inertia of airplanes whose size and weight differ widely. These are given in Table II and were obtained from the following expressions:

$$C_A = \frac{A}{\frac{W}{g}\,(h^2 + b^2)}$$

$$C_B = \frac{B}{\frac{W}{g}\,(h^2 + l^2)}$$

$$C_C = \frac{C}{\frac{W}{g}\,(l^2 + b^2)}$$

where

C_A, C_B, C_C = the coefficients for the moments of inertia about the X, Y, and Z axes, respectively,

A, B, C = the moments of inertia about the X, Y, and Z axes,

w = weight,

h = over-all height,

b = span,

l = over-all length.

Discussion

At times there is a necessity for knowing the moments of inertia of airplanes on which measurements have not been made. These can be estimated with the aid of design data, such as the balance diagram and weight analysis, by a summation of the moments of inertia of the individual parts; however, because of the inaccuracies in weights and dimensions, the accumulation of small errors may make the results unreliable. To check the accuracy of this method the moments of inertia of a PT-1 airplane were determined by computation and the results were compared with those obtained by direct measurement.

The actual weights of the various parts of the airplane were measured and were in good agreement with the weights given on the balance diagram. A three-view drawing was used in conjunction with the balance diagram to obtain the location of the component parts in relation to the reference axes of the airplane. The total moment of inertia about any axis was found by summing up the moments of inertia of the constituent parts about that axis. The smaller parts were assumed to be concentrated masses located at their centers of gravity, and the moment of inertia of each was taken as the mass times the square of the distance to the axis about which the moment of inertia was desired. For the larger units, such as wing panels and fuselage, account was taken of their dimensions and mass distribution. The wing panels were considered as bodies of uniform density and were integrated over their length. The moment of inertia of the fuselage was obtained by considering each piece of tubing separately. The engine, not easily divided into smaller units, was treated as a concentrated mass located at its center of gravity.

Calculations were made first with the major parts of the airplane treated as homogeneous masses and second with the major parts subdivided into smaller units. As was expected, the accuracy was greatly increased when the smaller units were used.

With the smaller subdivisions the moments of inertia were 20, 10, and 5 per cent below those obtained from swinging tests for the X, Y, and Z axes, respectively. The maximum accuracy could apparently be obtained with extremely small subdivisions; the labor involved, however, would be considerable and there is the possibility that, because of the additional errors introduced by increasing the number of measurements, the accuracy would not be greatly increased.

Occasionally a high degree of accuracy is not necessary; then, coefficients similar to those in Table II may be used to estimate the moments of inertia of somewhat similar airplanes. In using these coefficients it is only necessary to know the weight and over-all dimensions of the airplane considered. This method is probably as accurate as the one just described, and is obviously much more convenient. In fact, for the moment of inertia about the lateral axis, the present data indicate that an accuracy within 10 per cent can be obtained merely by using the average coefficient without regard to the structural characteristics of the airplane considered. For the same degree of accuracy, however, the coefficients for the other axes must be chosen with due consideration of the similarity between the airplane considered and the others for which the coefficients have been established.

The number of airplanes for which the coefficients are available will be added to from time to time as additional moment of inertia measurements are made. These coefficients should become especially helpful where swinging facilities are not available; however, the pendulum method must be resorted to when the utmost accuracy is desired.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., April 10, 1931.

Reference

1. Miller, M. P. : An Accurate Method of Measuring the Moments of Inertia of Airplanes. N.A.C.A. Technical Note No. 351, 1930.

TABLE I

Airplane	Type	Weight (lb.)	A (slug ft.^{2})	B (slug ft.^{2})	C (slug ft.^{2})	$K_X = \sqrt{A/\frac{W}{g}}$	$K_Y = \sqrt{B/\frac{W}{g}}$	$K_Z = \sqrt{C/\frac{W}{g}}$	Principal axes (angle with (X) body axis*)
VE-7	Naval Training Biplane Landplane	2208	1478	1498	2510	4.64	4.67	6.05	-3° 40'
PT-1	Army Training Biplane Landplane	2512	2205	2168	3343	5.31	5.26	6.54	
PW-9	Army Pursuit Biplane Landplane	2885	1400	1930	2673	3.95	4.63	5.46	-3° 30'
NY-1	Naval Training Biplane Landplane	2622	2336	2530	3887	5.35	5.56	6.90	-0° 15'
Doyle 0-2	Commercial Monoplane Landplane	1388	693	673	980	4.00	3.94	4.76	$+2^\circ$ 30'
02U-3	Naval Observation Biplane Landplane	3550	2817	2880	4530	5.05	5.11	6.41	$+0^\circ$ 35'
F4B-1	Naval Fighter Biplane Landplane	2540	1147	1590	2092	3.80	4.48	5.14	-0° 25'
NY-2	Naval Training Biplane Seaplane	2921	4477	3060	5688	7.02	5.80	7.91	$+0^\circ$ 40'
0-11	Army Observation Biplane Landplane	4558	2877	4238	6290	4.50	5.47	6.66	$+0^\circ$ 20'
NB-1	Naval Training Biplane Landplane	2544	2756	2306	4143	5.90	5.40	7.24	0°
XN2Y-1	Naval Training Biplane Landplane	1567	818	944	1316	4.09	4.40	5.19	0°

*Positive angles measured upward from tail of airplane.

TABLE II

Airplane	Over-all Dimensions Span (b)	Length (l)	Height (h)	$C_A = \frac{A}{M(b^2 + h^2)}$	$C_B = \frac{B}{M(l^2 + h^2)}$	$C_C = \frac{C}{M(b^2 + l^2)}$
VE-7	34.11	24.45	8.58	.0174	.0325	.0208
PT-1	34.79	27.67	9.00	.0218	.0329	.0217
PW-9	32.08	22.85	8.75	.0141	.0359	.0192
NY-1	34.48	27.75	9.67	.0223	.0359	.0243
Doyle O-2	30.00	19.00	7.92	.0168	.0367	.0180
O2U-3	34.50	24.63	10.04	.0198	.0366	.0228
F4B-1	30.00	20.61	9.58	.0146	.0389	.0200
NY-2	40.00	27.75	11.74	.0283	.0371	.0264
O-11	38.00	28.33	10.08	.0131	.0330	.0197
NB-1	36.83	25.17	10.73	.0236	.0390	.0263
XN2Y-1	28.00	20.75	7.83	.0198	.0394	.0222

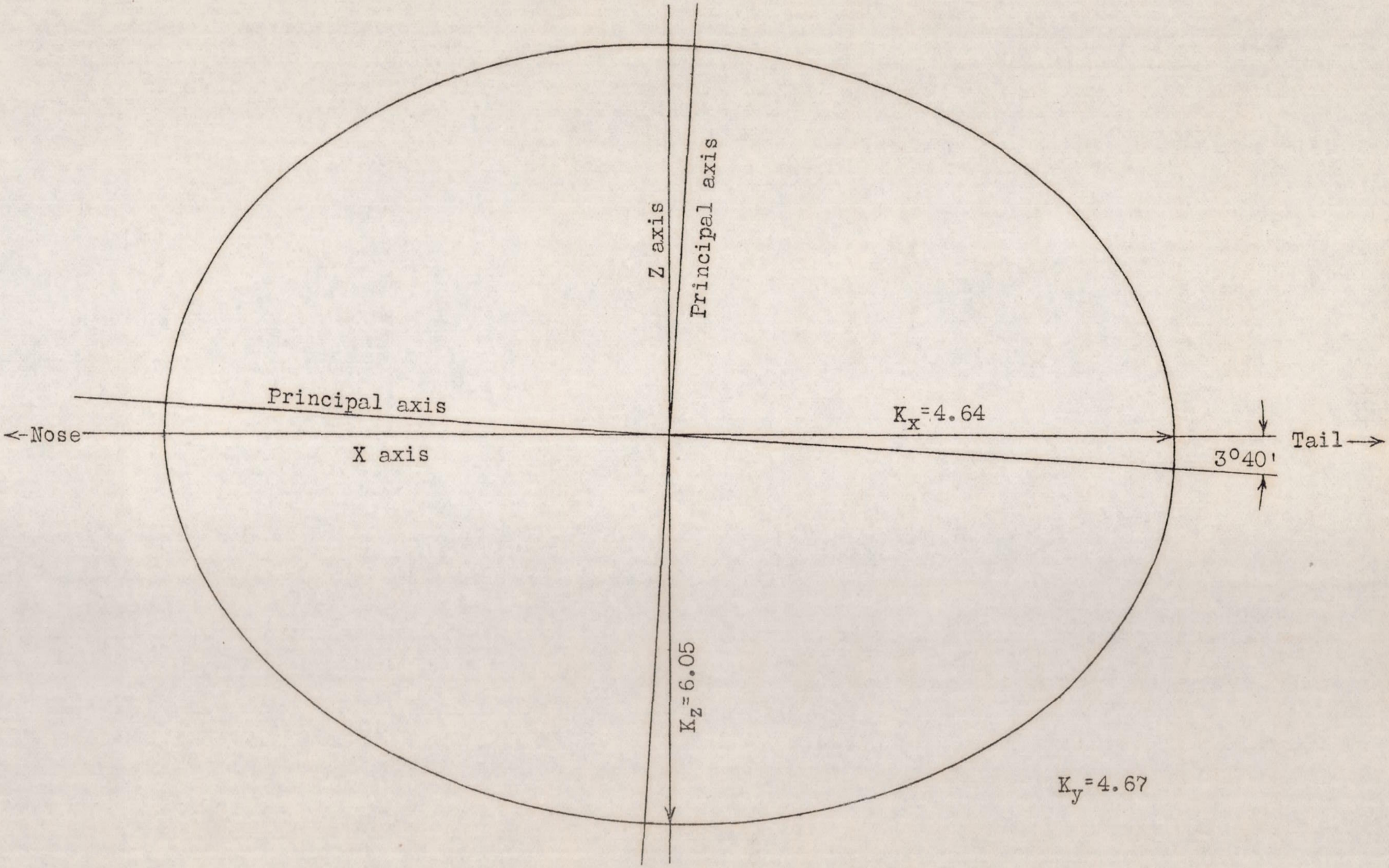

Fig. 1 Ellipse of inertia. VE-7